Fréquences spatiales

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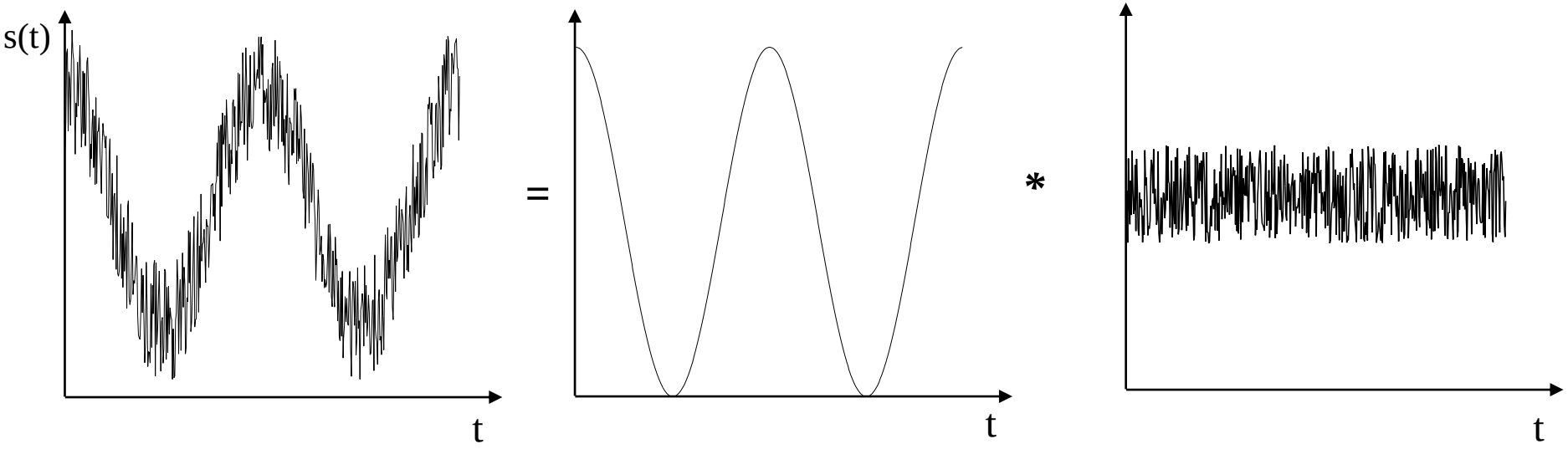
Fréquences temporelles

Signal temporel 1D:

Hautes fréquences: *fortes variations de signal sur petite durée*

Basses fréquences: *fortes variations de signal sur grande durée*

basse fréquence *haute fréquence*



Fréquences spatiales

Image (signal spatial 2D):

Hautes fréquences: *fortes variations de CN sur petite distance (contours, détails,…)*

Basses fréquences: *variations de CN sur grande distance (dégradés de fond d'image,…)*

Transformée de Fourier 2D

f(x,y) *image analogique*

Transformée de Fourier: $$F(u,v)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\, e^{[-2j\pi(ux+vy)]}dxdy$$

u, ***v:*** *fréquences spatiales*

Transformée de Fourier inverse: $$f(x,y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} F(u,v)\, e^{[2j\pi(ux+vy)]}\, dudv$$

Notation polaire: $F(u,v)=\left|F(u,v)\right|e^{j\varphi(u,v)}$

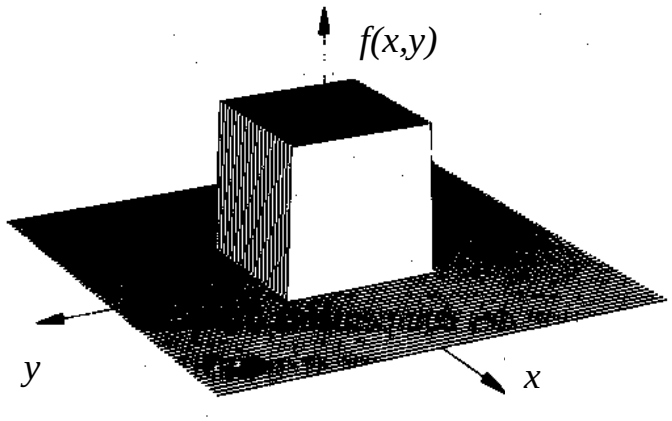
Spectre d'amplitude: $\left|F(u,v)\right|$

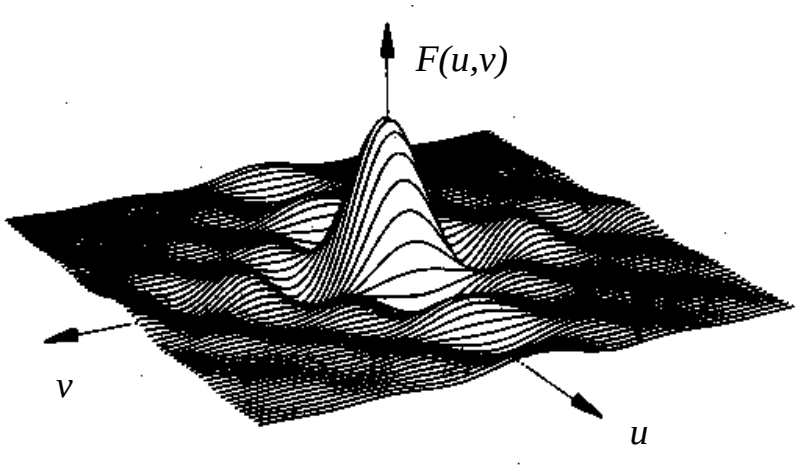
Spectre de puissance: $\left|F(u,v)\right|^2$

Spectre de phase: $\varphi(u,v)$

Exemple

$$f(x,y) = A \begin{cases} -X/2 \le x \le X/2 \\ -Y/2 \le y \le Y/2 \end{cases}$$

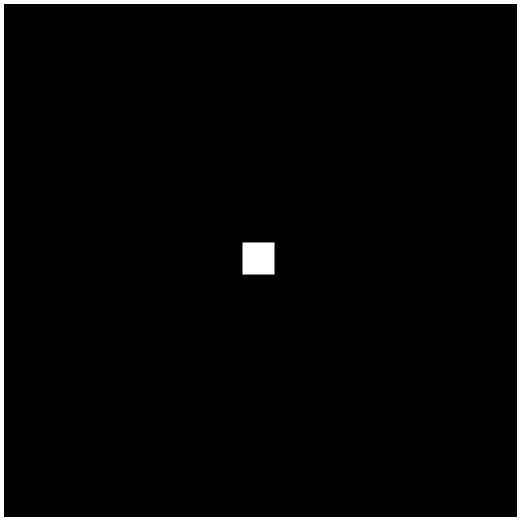


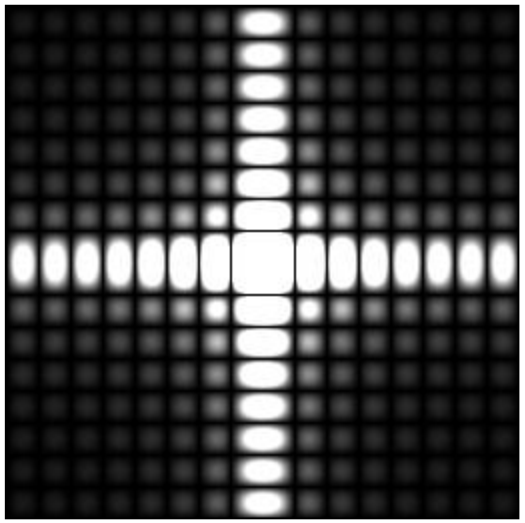


$$F(u,v) = AXY\,\mathrm{sinc}(\pi uX)\,\mathrm{sinc}(\pi vY)$$

Exemple

$$f(x,y) = A \quad \begin{cases} -X/2 \le x \le X/2 \\ -Y/2 \le y \le Y/2 \end{cases}$$





$$\left|F(u,v)\right| = \left|AXY\ \mathrm{sinc}(\pi uX)\,\mathrm{sinc}(\pi vY)\right|$$

Transformée de Fourier 2D discrète

I(m,n) *image numérique* $\quad \boldsymbol{n} = 0, 1, 2, \ldots M\text{-}1$

$\boldsymbol{m} = 0, 1, 2, \ldots N\text{-}1$

Transformée de Fourier Discrète:

$$F(k,l) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I(m,n)\, e^{\left[-2j\pi\left(k\frac{m}{M}+l\frac{n}{N}\right)\right]}$$

k, l: *fréquences spatiales*

Transformée de Fourier inverse:

$$I(m,n) = \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} F(k,l)\, e^{\left[2j\pi\left(m\frac{k}{M}+n\frac{l}{N}\right)\right]}$$

pas de l'échantillonage spatial: Δx *et* Δy

$$x \in [0; M\Delta x]$$
$$y \in [0; N\Delta y]$$

Pas de l'échantillonage fréquentiel: $\quad \Delta u = \frac{1}{M\Delta x} \quad$ et $\quad \Delta v = \frac{1}{N\Delta y}$

$$u \in [0; 1/\Delta x]$$
$$v \in [0; 1/\Delta y]$$

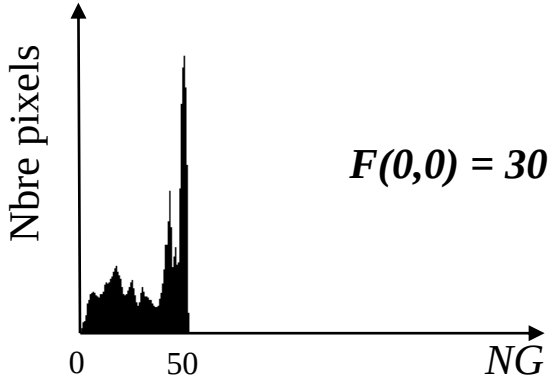
Propriété de la TF 2D:

$$F(k,l) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I(m,n)\, e^{\left[-2j\pi\left(k\frac{m}{M}+l\frac{n}{N}\right)\right]}$$

☞ ***Moyenne de l'image:***

$$F(0,0) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I(m,n)$$

Image originale I



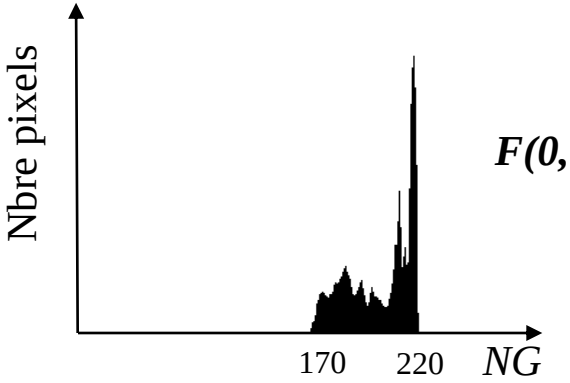
F(0,0) = 30

F = FFT(I)

F(0,0) = 200

R = FFT^{-1}(F)

Image résultat R

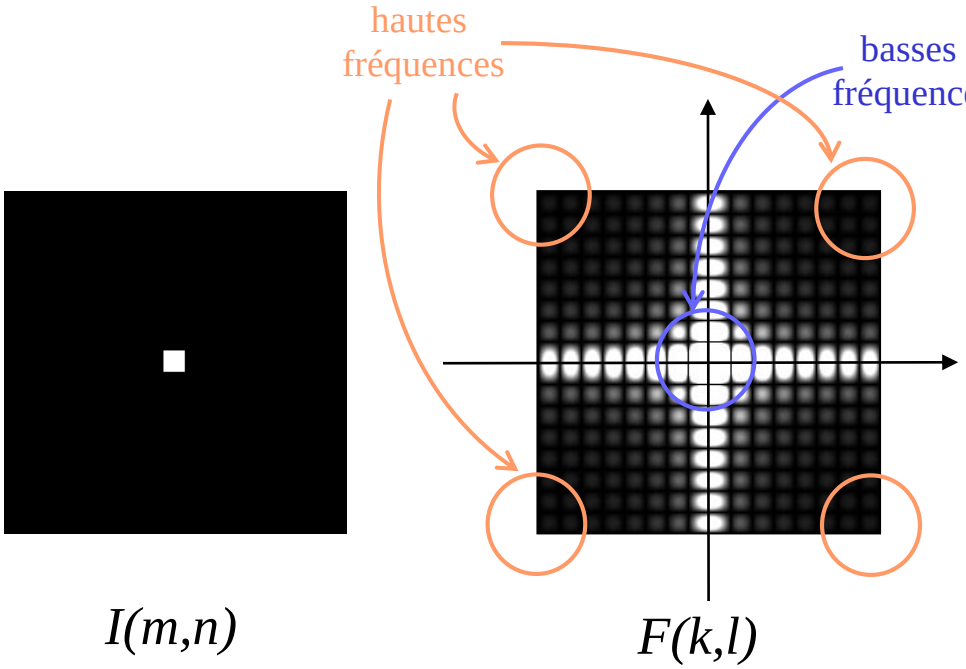


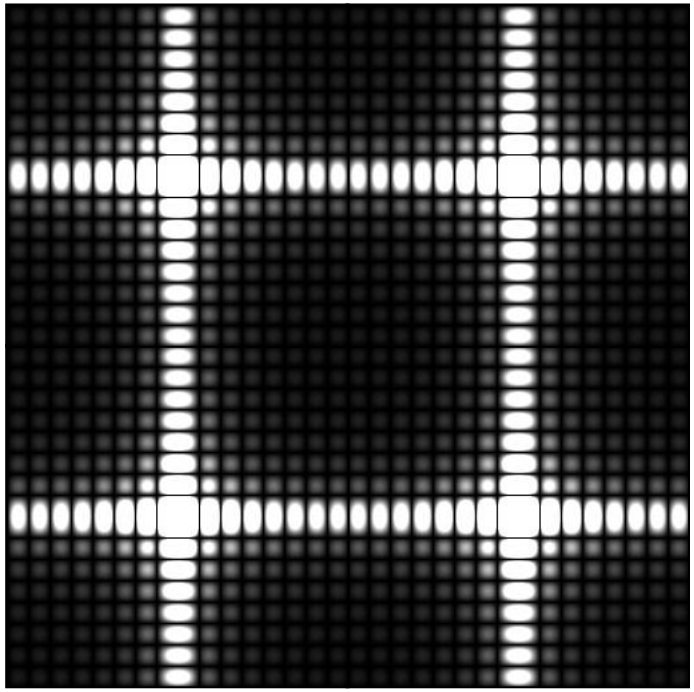
F(0,0) = 200

Propriétés de la TF 2D discrète

$$F(k,l) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I(m,n)\, e^{\left[-2j\pi\left(k\frac{m}{M}+l\frac{n}{N}\right)\right]}$$

☞ *Fonction* ***périodique*** *de période (M, N)*

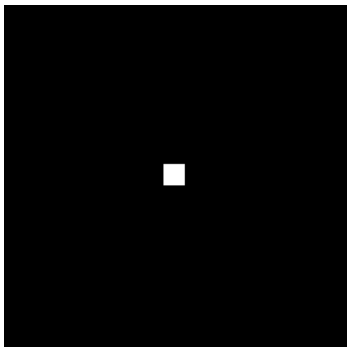




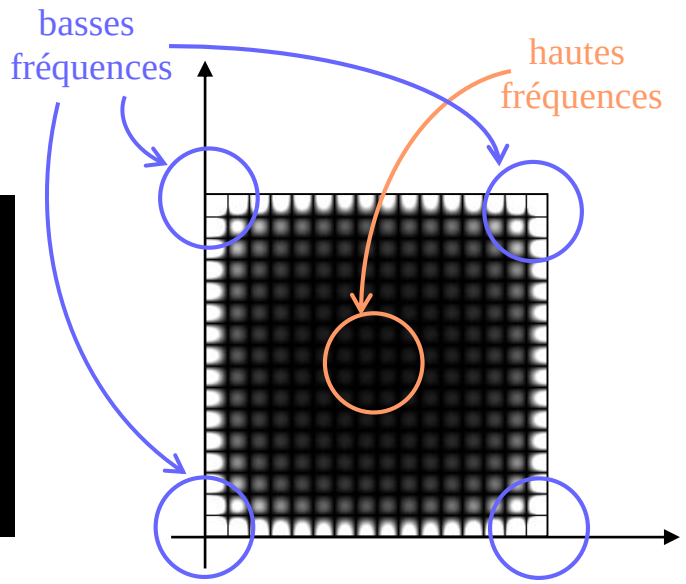
Propriétés de la TF 2D discrète

$$F(k,l) = \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} I(m,n)\, e^{\left[-2j\pi\left(k\frac{m}{M}+l\frac{n}{N}\right)\right]}$$

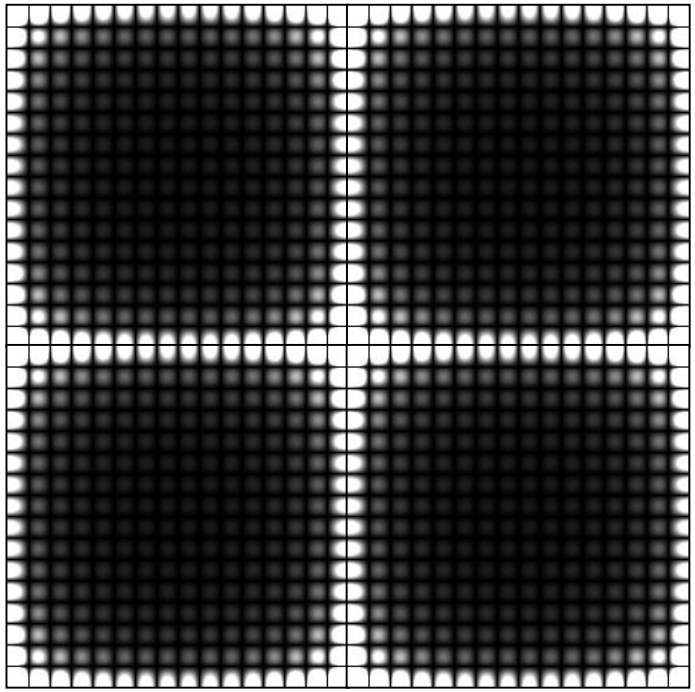
☞*Fonction* ***périodique*** *de période (M, N)*



I(m,n)



F(k,l)



☞*Représentation usuelle dans les logiciels standards*

Propriétés TF 2D discrète

☞ ***Séparabilité***

$$F(k,l) = \frac{1}{M\,N}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} I(m,n)\, e^{\left[-2j\pi\left(k\frac{m}{M}+l\frac{n}{N}\right)\right]}$$

$$= \frac{1}{M}\sum_{m=0}^{M-1}\left(\frac{1}{N}\sum_{n=0}^{N-1} I(m,n)\, e^{\left(-2j\pi l\frac{n}{N}\right)}\right) e^{-2j\pi k\frac{m}{M}}$$

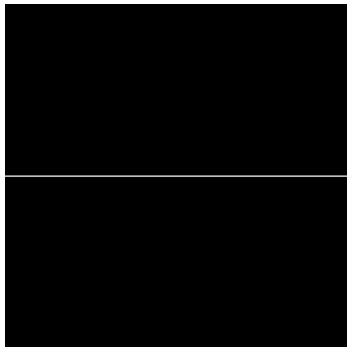
$$= \frac{1}{M}\sum_{m=0}^{M-1} I(m,l) e^{\left(-2j\pi k\frac{m}{M}\right)}$$

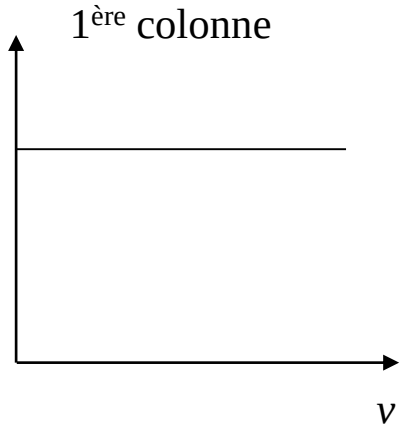
☞ *Calcul d'une TF-1D sur chaque colonne, puis TF-1D sur chaque ligne*
ou
☞ *Calcul d'une TF-1D sur chaque ligne, puis TF-1D sur chaque colonne*

EXEMPLES

I(m,n)

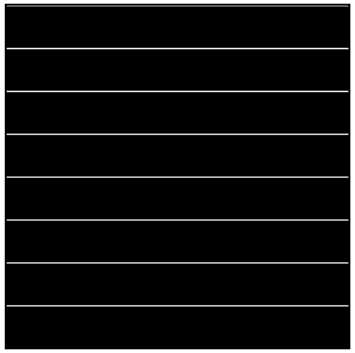
$|F(u,v)|$

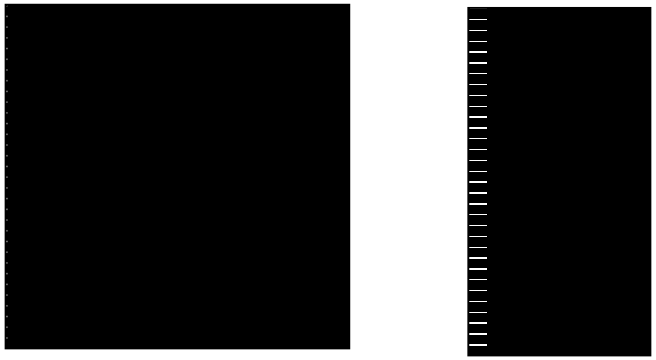


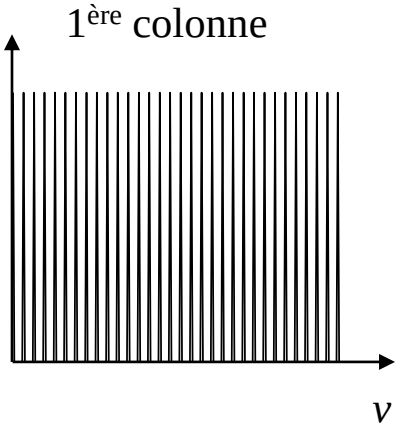


I(m,n)

$|F(u,v)|$



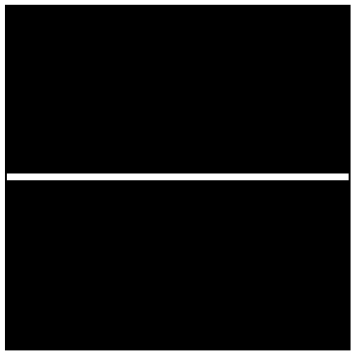


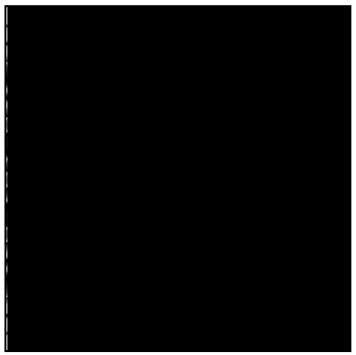


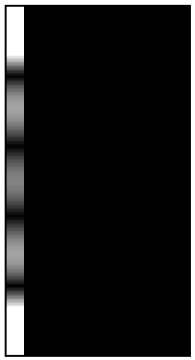
EXEMPLES

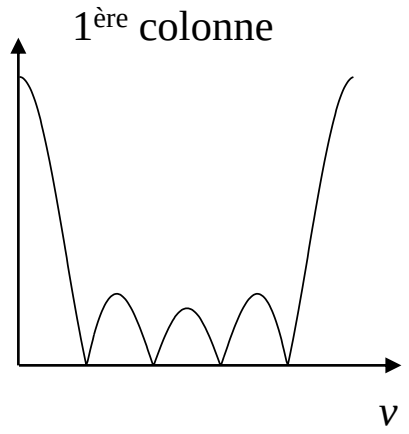
I(m,n)

$|F(u,v)|$



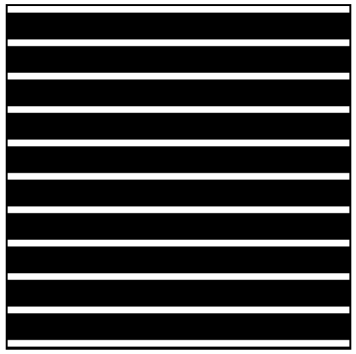


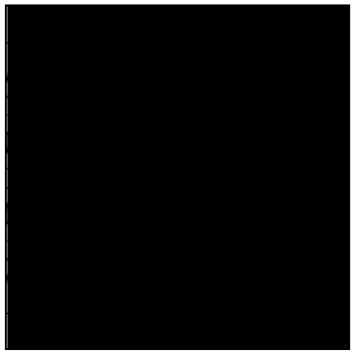


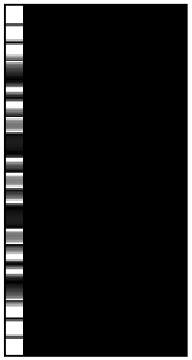


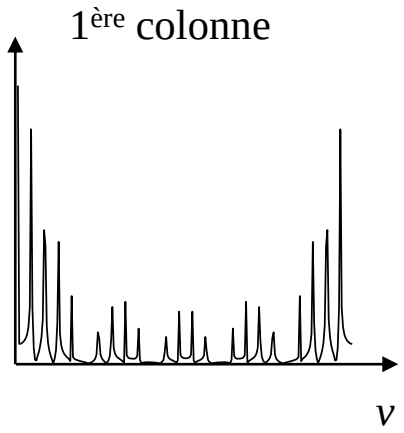
I(m,n)

$|F(u,v)|$



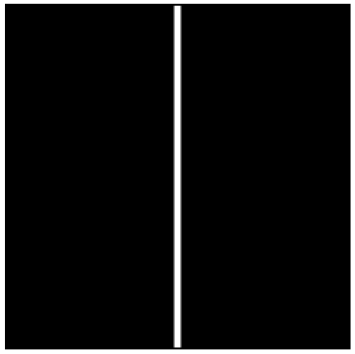




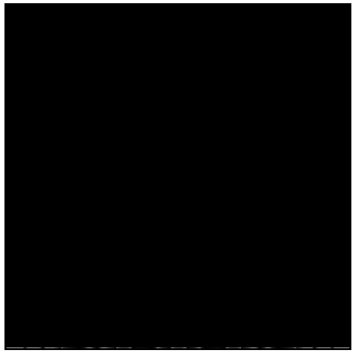


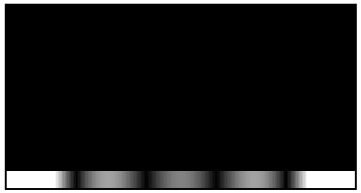
EXEMPLES

I(m,n)

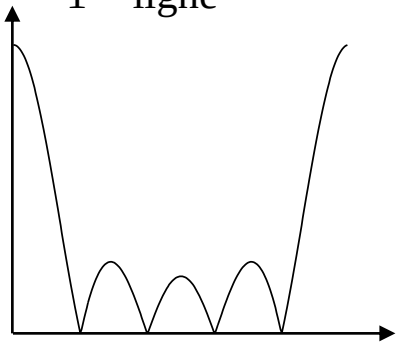


$|F(u,v)|$





1ère ligne



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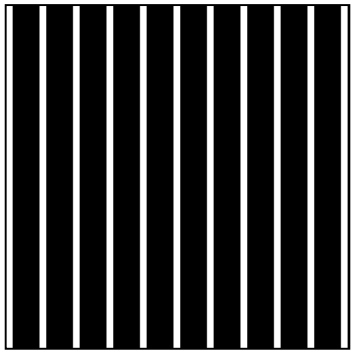
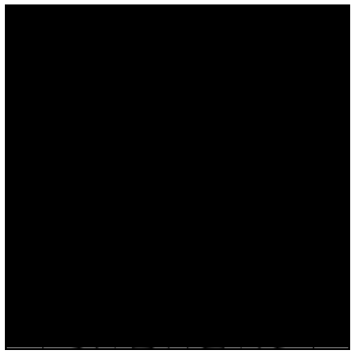
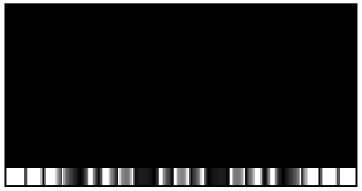
I(m,n)

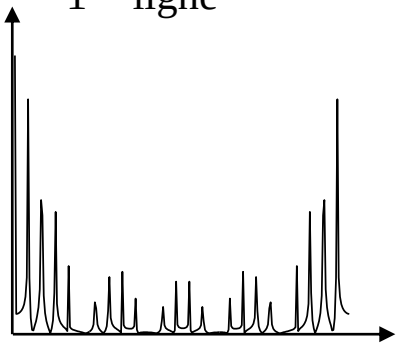


$|F(u,v)|$





1ère ligne



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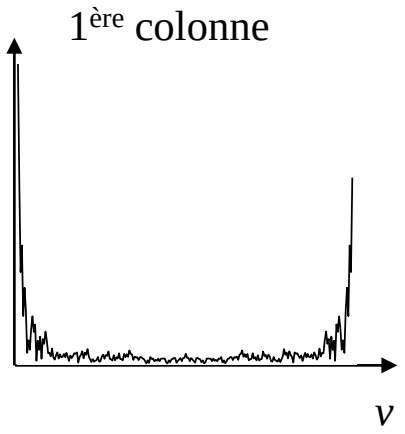
EXEMPLES

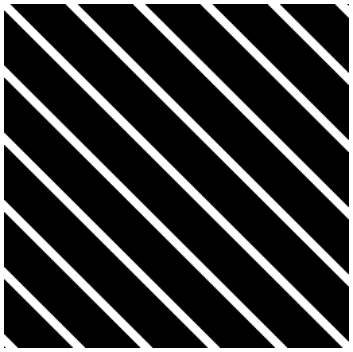
I(m,n)

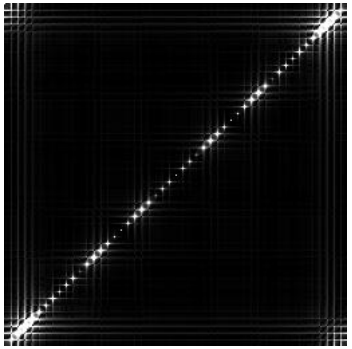
$|F(u,v)|$

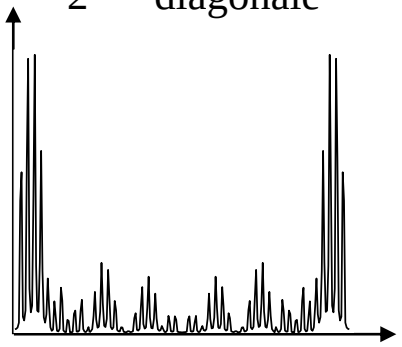


I(m,n)

$|F(u,v)|$

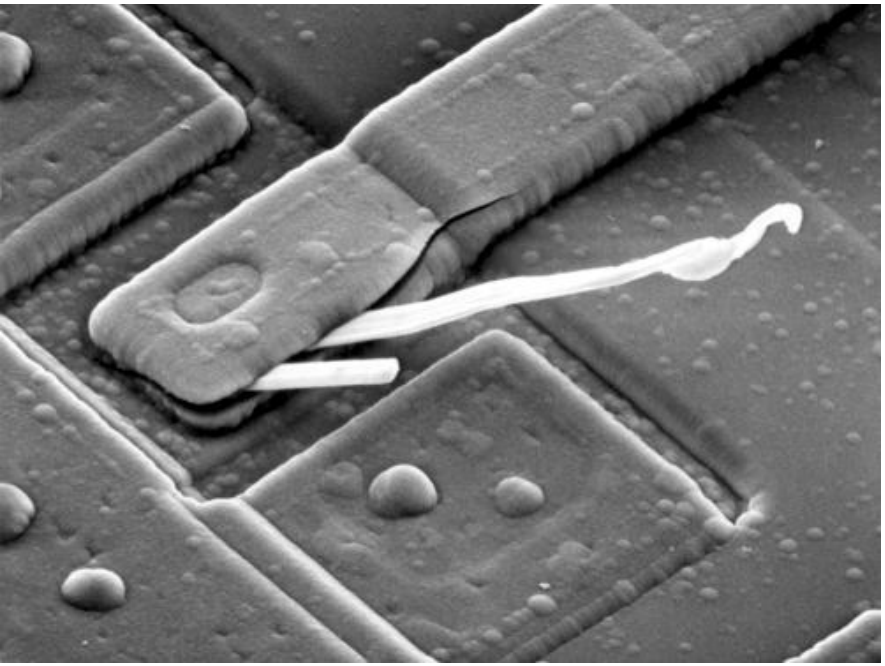




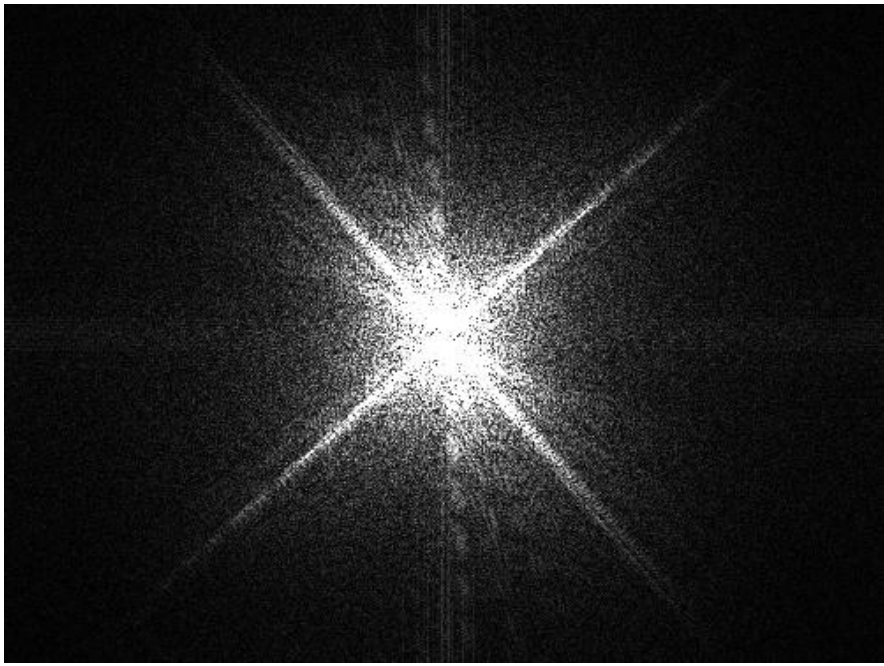


EXEMPLES

I(m,n)



$|F(u,v)|$



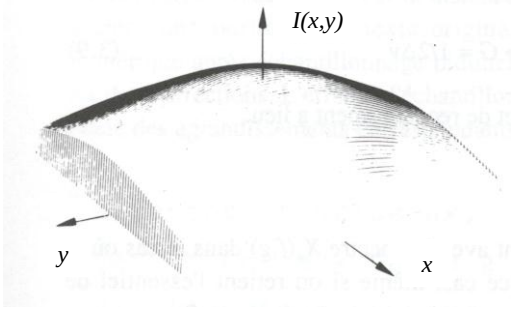
Source: Gonzalez & Woods

Echantillonnage 2D

$$F(u,v) = TF[I(x,y)] \quad \begin{matrix} |u| \le U_M \\ |v| \le V_M \end{matrix}$$

condition de Shannon: $\Delta u = \frac{1}{T_{xe}} > 2U_M \quad ; \quad \Delta v = \frac{1}{T_{ye}} > 2V_M$

image analogique: I



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Peigne 2D: p

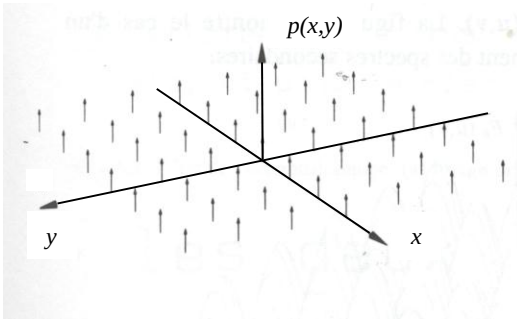

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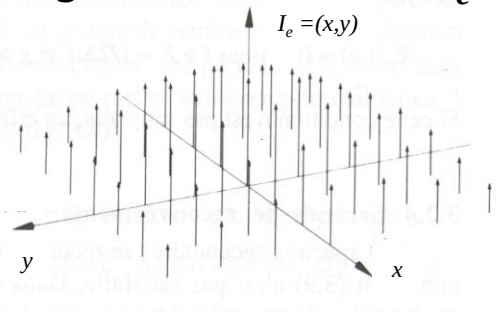
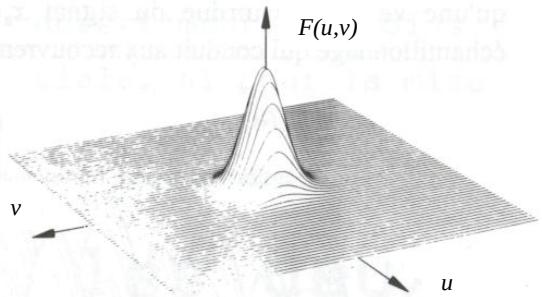
image échantillonnée: I_e



Spectre de I



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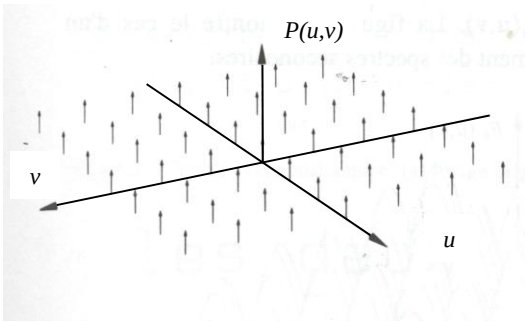
Spectre de p



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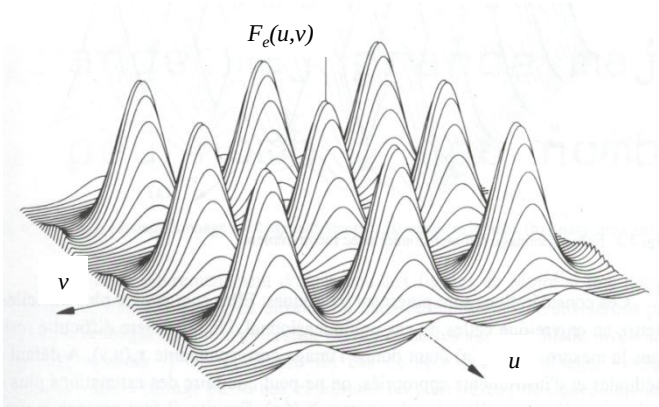
Spectre de I_e

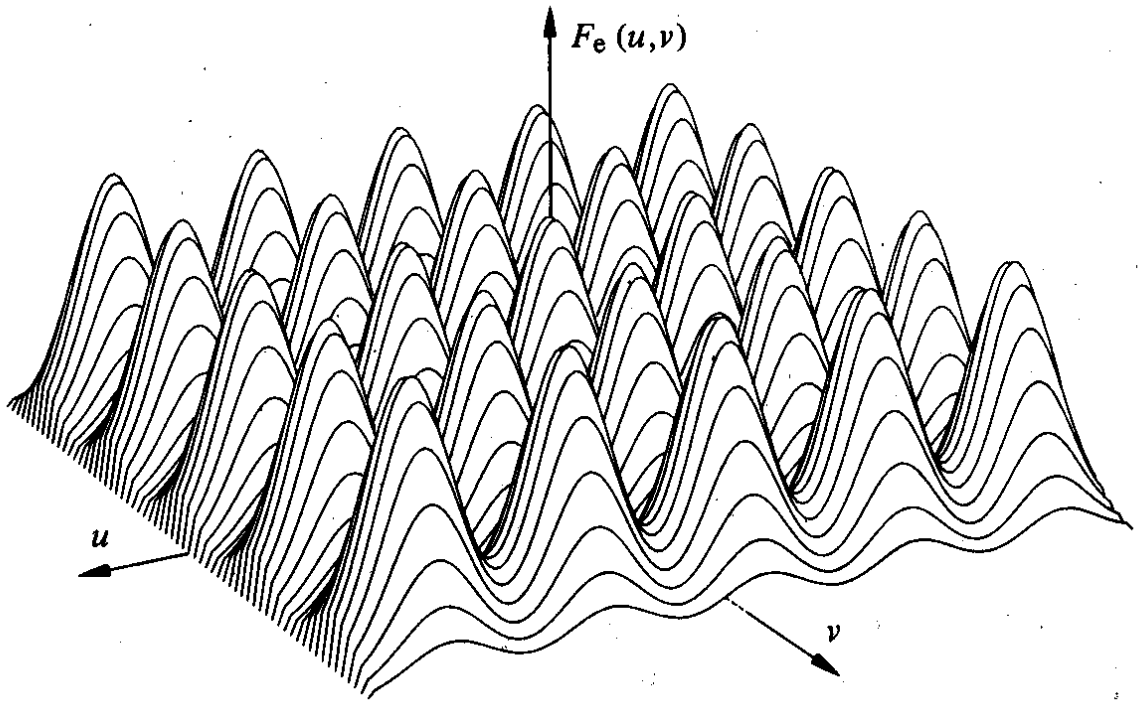


échantillonnage sans recouvrement de spectre (Shannon respecté)

Source: Kunt

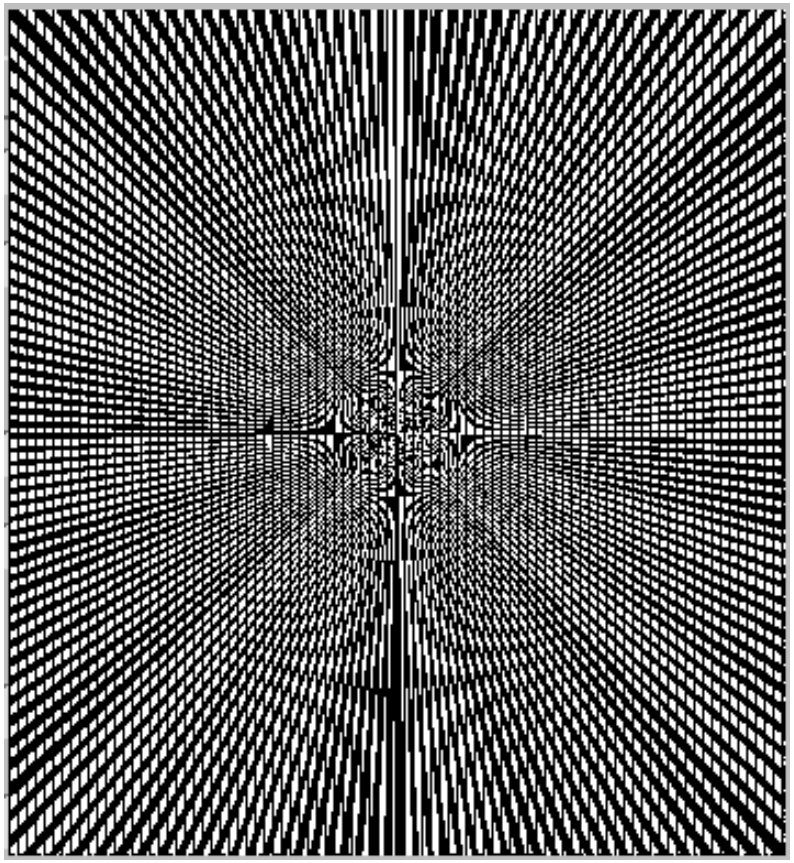
Echantillonnage 2D

Recouvrement de spectre (Shannon non respecté)



Source: Kunt

Exemple: Figure de Moiré

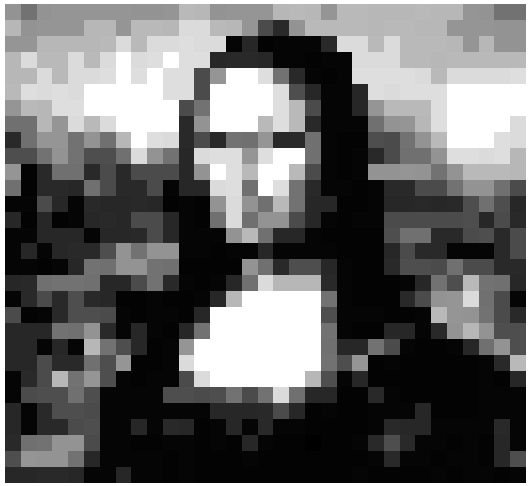


☞ Remède: Suppression des hautes fréquences avant échantillonnage (filtre passe-bas)

Sélection 1 pixel / bloc (4x4)

Sélection 1 pixel / bloc (8x8)

Non respect de Shannon



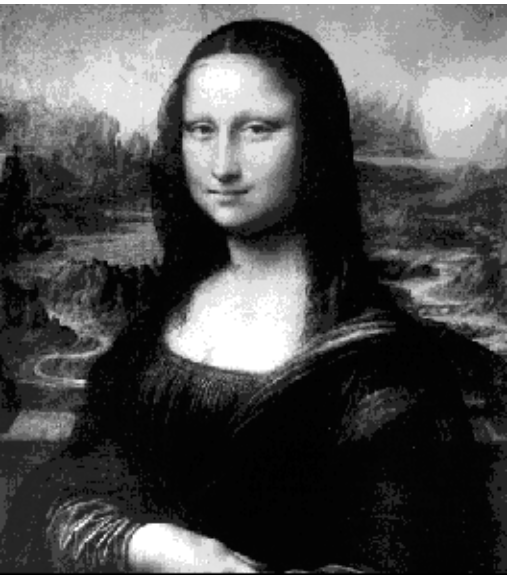


image originale

Moyennage par bloc 4x4 pixels

Moyennage par bloc 8x8 pixels

Application d'un filtre passe-bas