RADIOMETRY

Study of the radiation power

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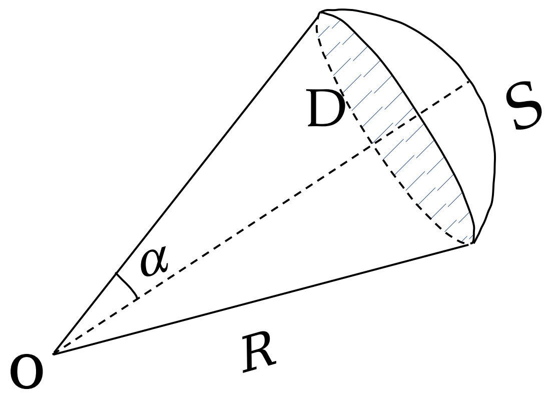
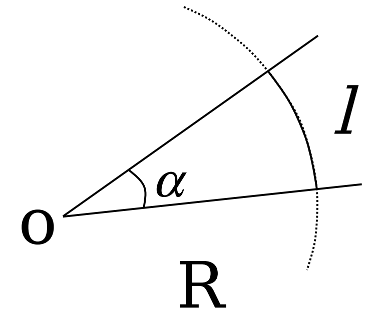
Solid angle (3D)



Plane angle (2D)

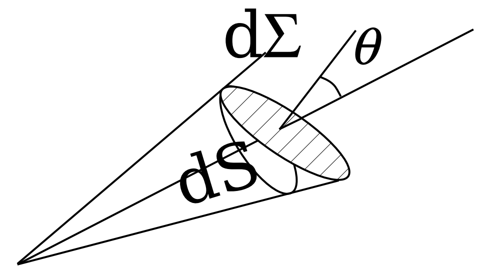


$$\alpha = \frac{l}{R}$$

Stéradians (sr)

$$\Omega = \int d\Omega = \int \frac{dS}{R^2} = \frac{R^2}{R^2} \int_{\theta=0}^{\alpha} \int_{\varphi=0}^{2\pi} \sin\theta \, d\theta \, d\varphi$$

$$\Omega = 2\pi(1 - \cos\alpha)$$



Si Ω petit:

$$\Omega \approx \frac{D}{R^2} \approx \pi\alpha^2$$

$$d\Omega = \frac{dS}{R^2} = \frac{d\Sigma \cos\theta}{R^2}$$

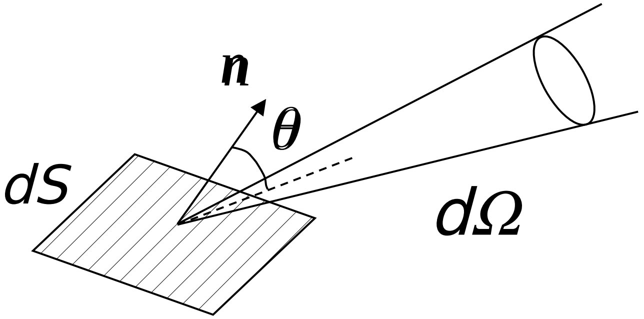
Radiometric quantities

tion power: Radiant power emited by the source along the light rays

$$\Phi = \frac{dQ}{dt} \qquad (W)$$

ant flux emited by the elementary surface source dS,
e solid angle dΩ around the direction θ :

$$\delta^2\Phi = L \cos\theta \, dS \, d\Omega$$



radiance ($W.m^{-2}sr^{-1}$)

L is θ independent, the source is called ***lambertian***

Radiometric quantities (2)

nsity *of a source:* Radiant Flux / Solid angle unit

$$I = \frac{d\Phi}{d\Omega} = \int_{Source} L \cos\theta \, dS \qquad (\mathrm{W.sr^{-1}})$$

ance *of a source: Radiant f*lux / Surface unit

$$M = \frac{d\Phi}{dS} = \int_{h\acute{e}m.} L \cos\theta \, d\Omega \qquad (\mathrm{W.m^{-2}})$$

lambertian source:

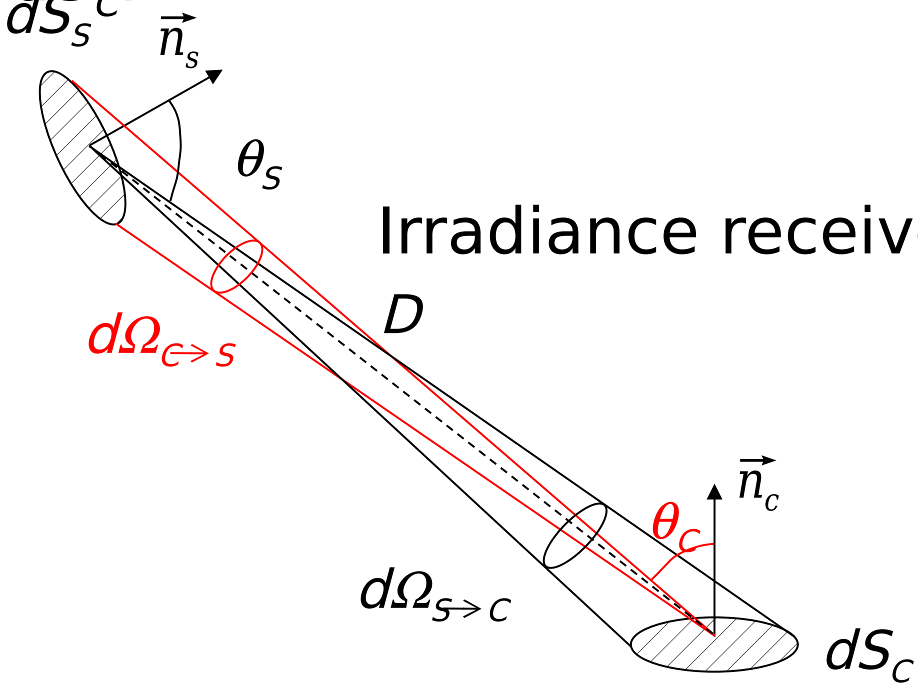
$$M = L \int_{h\acute{e}m.} \cos\theta \, d\Omega = \pi L$$

urface is lightened (not a source): ***Irradiance*** *(instead of E*

Radiometric quantities (3)

diance *received on dS_C highlighted by the source dS_S:*

Recived power dS_C:



¿ $L_s \cos\theta_t \, dS_c \, d\Omega_{c\rightarrow s}$

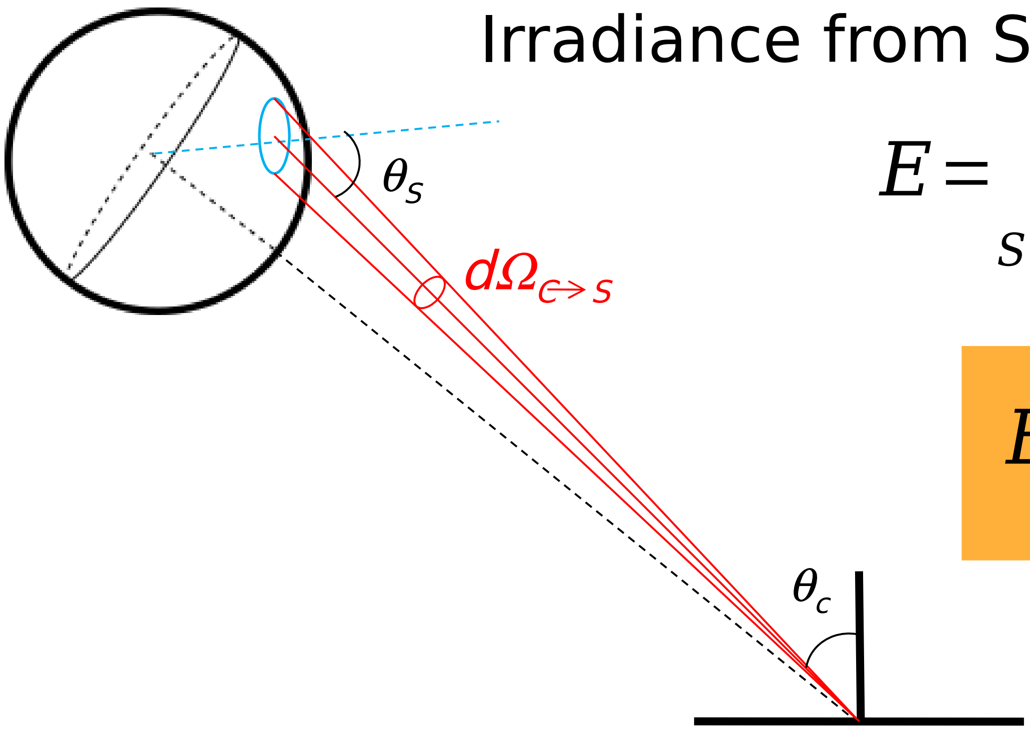
Irradiance received by dS_C:

¿ $L_s \cos\theta_c \, d\Omega_{c\rightarrow s}$

Radiometric quantities (4)

Irradianc e from dS_s:

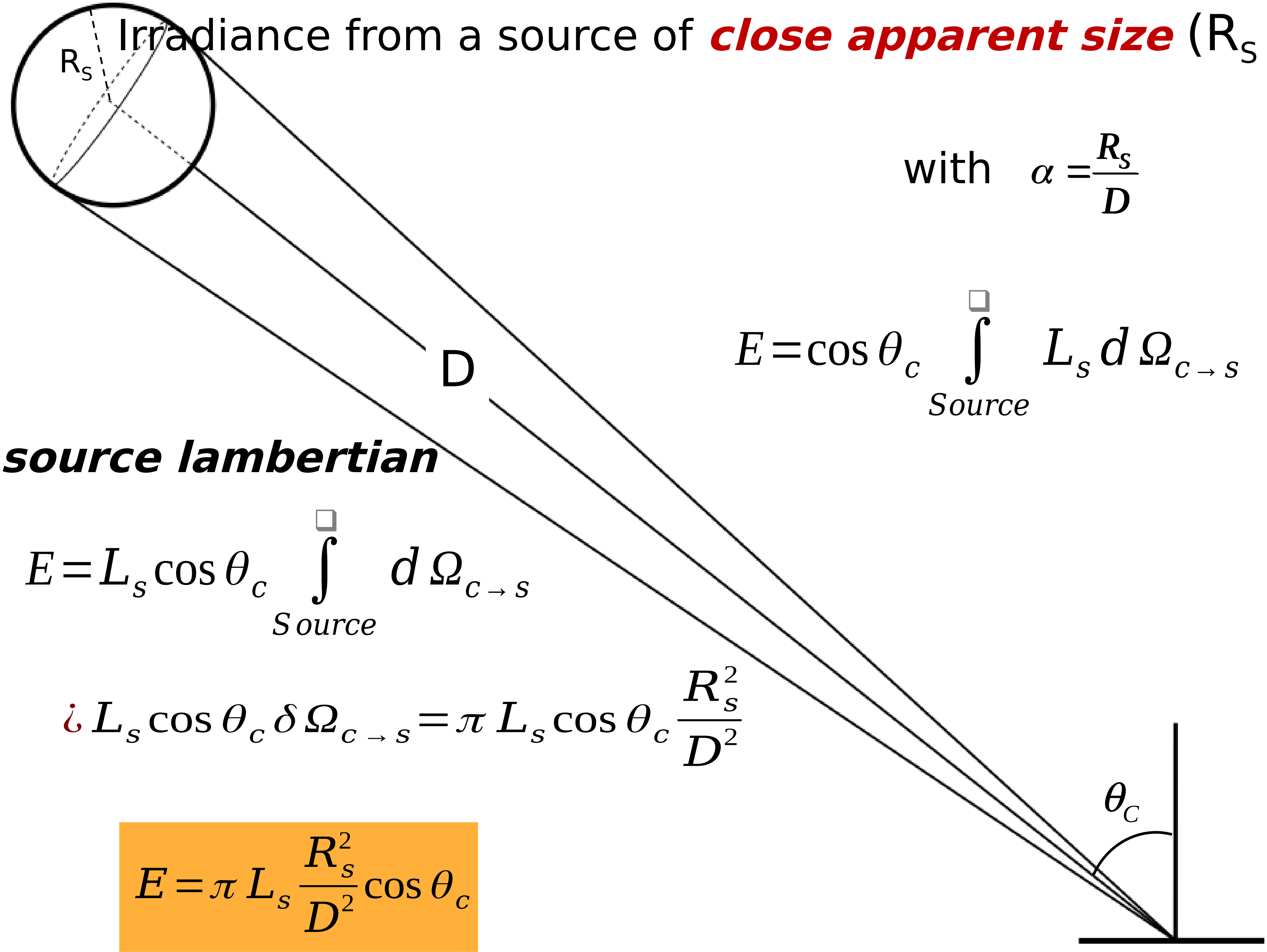
Irradiance from S:



$$E = \int_{Source} \frac{L_s \cos\theta_s \, dS_s \cos\theta_t}{D^2}$$

$$E = \int_{Source} L_s \cos\theta_t \, d\Omega_{t\to s}$$

Irradiance from a source of ***close apparent size*** (R_s



with $\alpha = \frac{R_s}{D}$

$$E = \cos\theta_c \int_{Source} L_s \, d\Omega_{c \to s}$$

source lambertian

$$E = L_s \cos\theta_c \int_{Source} d\Omega_{c \to s}$$

$$¿\, L_s \cos\theta_c \, \delta\Omega_{c \to s} = \pi L_s \cos\theta_c \frac{R_s^2}{D^2}$$

$$E = \pi L_s \frac{R_s^2}{D^2} \cos\theta_c$$

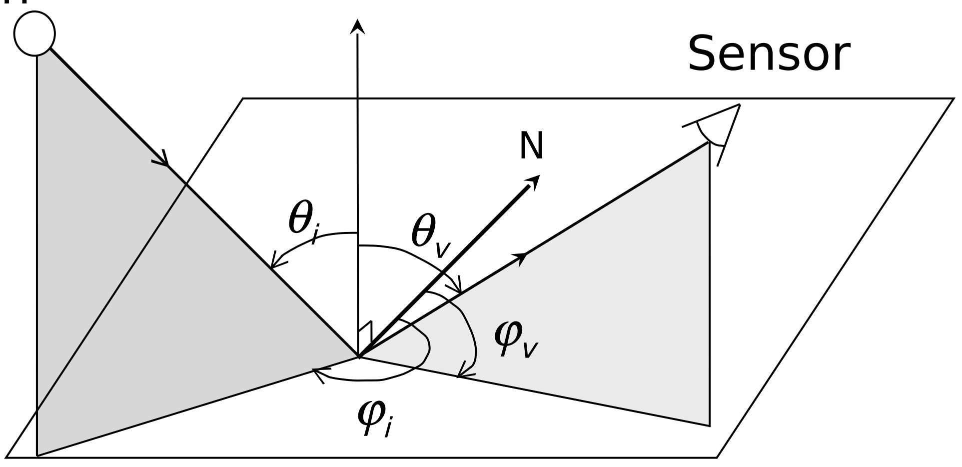
Optical measurements (0.4 – 5 µm)

(Reflection of Solar Radiation)

éflectance*:* characterize the studied surface

Bidirectionnal réflectance :

$$\rho(\theta_i,\varphi_i,\theta_v,\varphi_v,\lambda)=\frac{L_r}{E_i}=\frac{L_r}{L_i\cos\theta_i d\Omega_i}$$



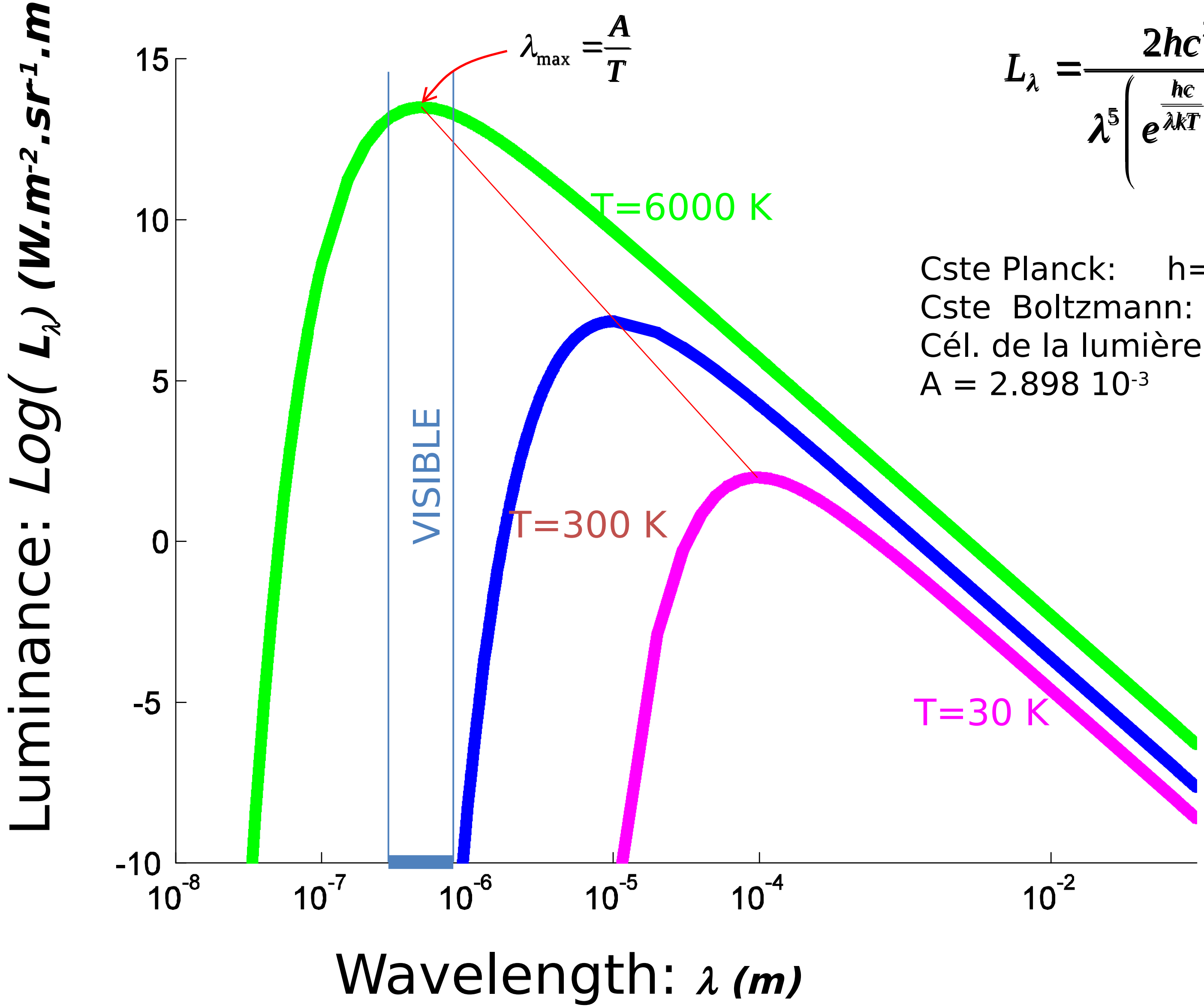
Albedo: $$a=\frac{\int_{hém.} L_r\cos\theta_v d\Omega_v}{\int_{hém} L_i\cos\theta_i d\Omega_i}=\frac{M}{E_i}$$

Reflectance Factor:

$$\rho_b=\frac{\rho_r}{\rho_r^{ref}}=\frac{L_r}{L_r^{ref}}=\frac{\pi L_r}{E_i}$$ avec $$E_i=L_{sol}\frac{\pi R_{sol}^2}{D_{ST}^2}\cos\theta_i$$ $\Rightarrow$ $$\boxed{\rho_b=\frac{1}{L_{sol}R_{sol}^2}D_{ST}^2\frac{L_r}{\cos\theta'}}$$

Black body radiation

body: Ideal body in theromdynamic equilibrium with its environment.
It absorbs totally any incoming radiation and emits maximum radiation at all wavelentg
ty: **lambertian**



$$\lambda_{max} = \frac{A}{T}$$

$$L_\lambda = \frac{2hc^2}{\lambda^5\left(e^{\frac{hc}{\lambda kT}} - 1\right)}$$

Cste Planck: $h=6.62\ 10^{-34}$ SI
Cste Boltzmann: $k=1.38\ 10^{-23}$ SI
Cél. de la lumière: $c= 3\ 10^{8}\ m.s^{-1}$
$A = 2.898\ 10^{-3}$

Radiometric quantities

Integrated quantities *

Radiation Flux: $\Phi = \frac{dQ}{dt}$ *(W)*

Exitance ***M*** (W.m^{-2})

Irradiance ***E*** (W.m^{-2})

Intensity ***I*** (W.sr^{-1})

Radiance: **L** (W.m^{-2}.sr^{-1})

er the whole or part of the
ectromagnetic spaectrum

Spectral quantities**

Spectral flux: $\Phi = \frac{dQ}{dt}$ *(W.m^{-1})*

Spectral exitance ***M*** (W.m^{-2}.m^{-1})

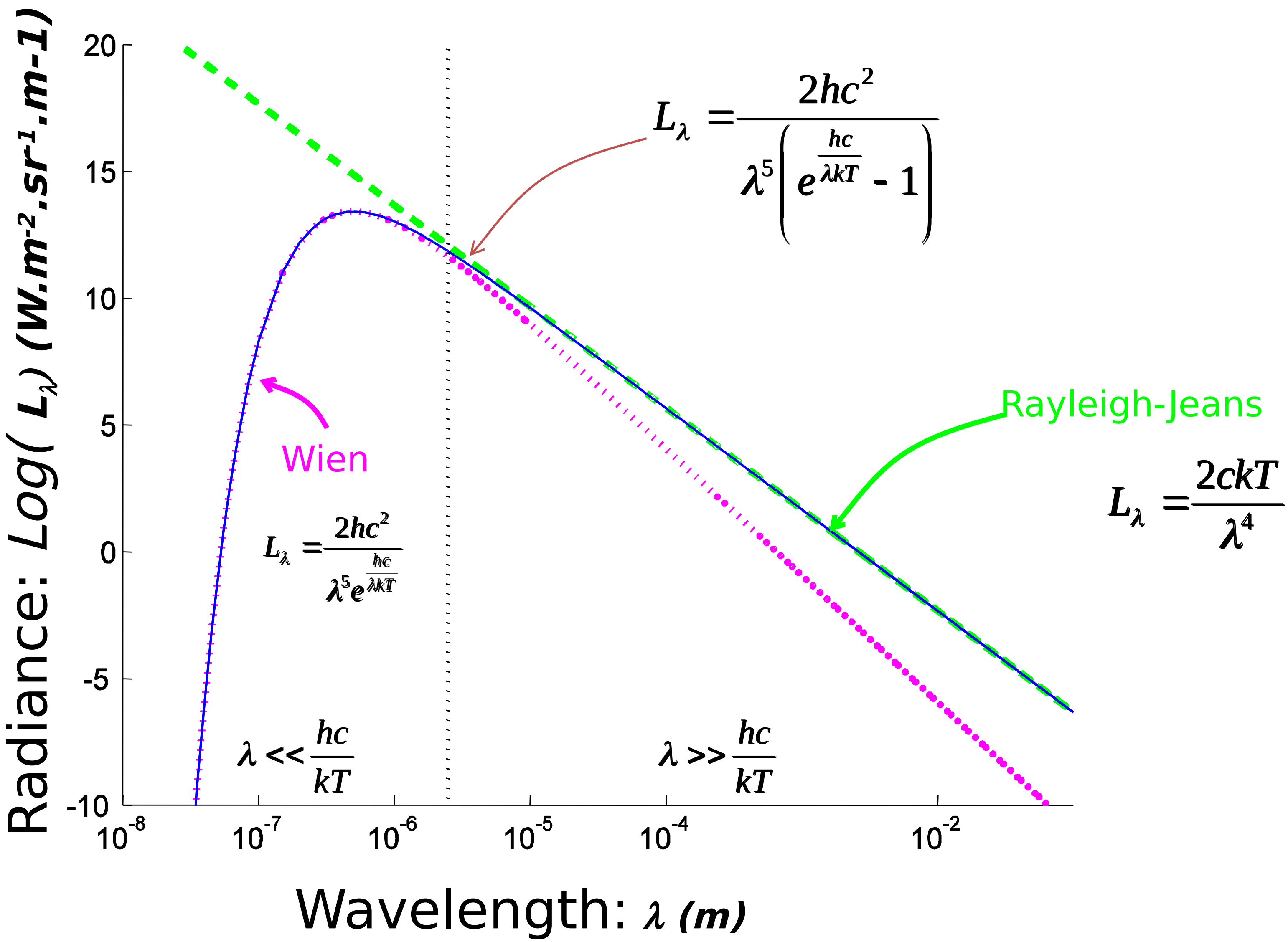
Spectral irradiance ***E*** (W.m^{-2}.m^{-1})

Spectral intensity ***I*** (W.sr^{-1}.m^{-1})

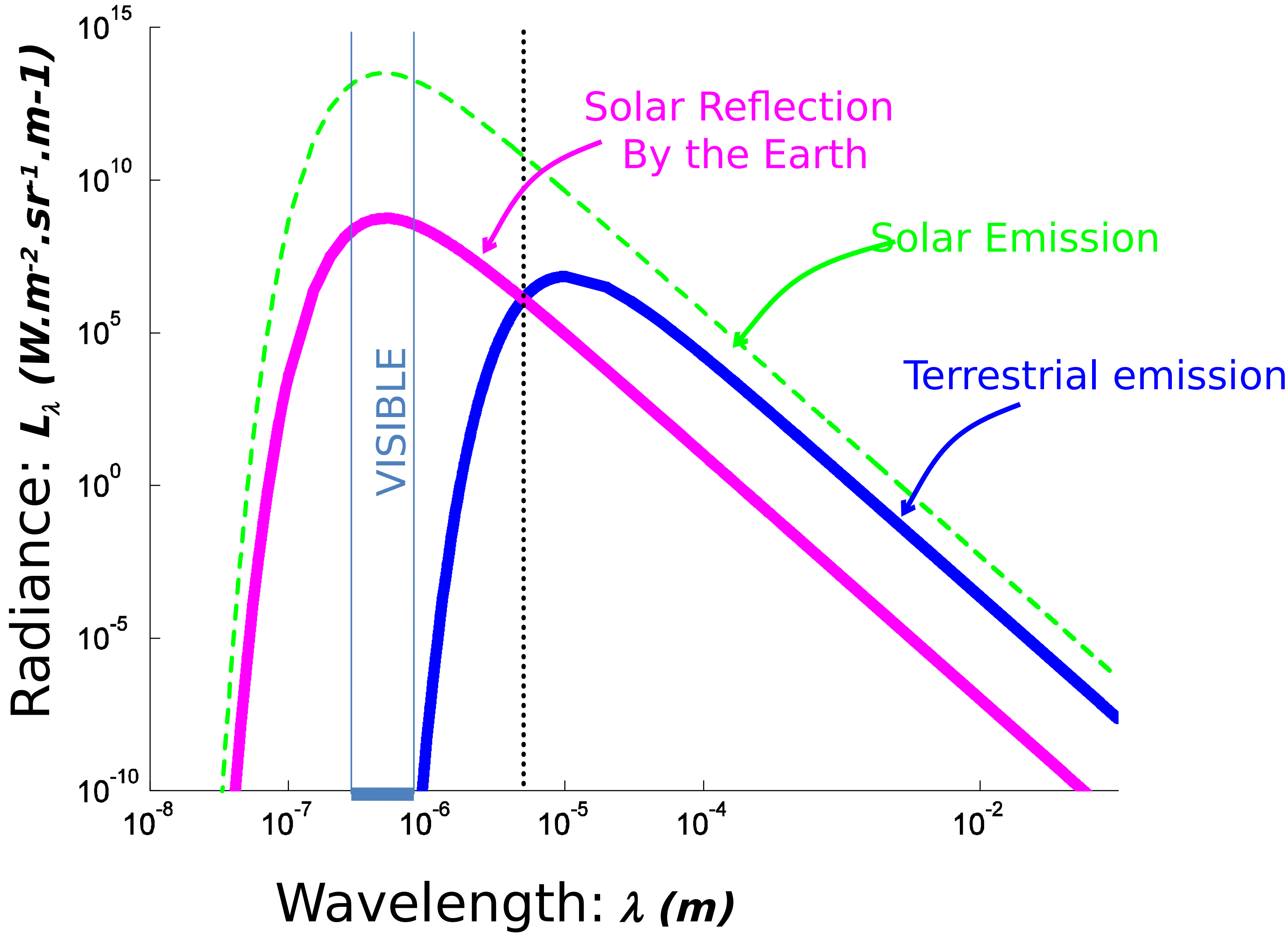
Spectral radiance ***L*** (W.m^{-2}.sr^{-1}.m^{-1})

** For a given wavelength
Sometimes, µm or le nm is preferd
than m for the unit associated to the

Black body radiation: Wien and Rayleigh-Jeans approximations



The electromagnetic radiation Coming from the Earth



rmal IR+ passive microwaves (5 μm – 10 m)

mited radiations by the surfaces)

Black Body(ideal): $L_\lambda = \frac{2ckT}{\lambda^4}$

Radiance of the studied body

Radiance of the equivalent blackbody having the same physical temperature

Gray Body(actual) $L_\lambda = \varepsilon(\lambda)\, L_\lambda$

cn

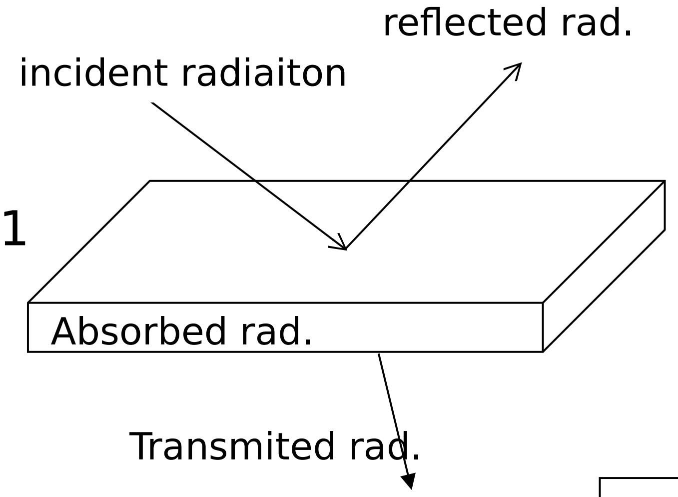
$0 \leq \varepsilon(\lambda) \leq 1$

ess temperature T_b: *physical temperature of the black body tha emit the same radiation than the sutdied body*

$$\frac{2ckT_b}{\lambda^4} = \varepsilon \frac{2ckT}{\lambda^4} \quad \Rightarrow \quad T_b = \varepsilon\, T$$

Energy conservation

$$0 \le \rho_\lambda, \alpha_\lambda, \tau_\lambda \le 1$$



reflectance $\rho_\lambda = \dfrac{radiation \quad réfléchie}{radiation \quad incidente}$

absorptance $\alpha_\lambda = \dfrac{radiation \quad absorbée}{radiation \quad incidente}$

transmittance $\tau_\lambda = \dfrac{radiation \quad transmise}{radiation \quad incidente}$

$$\rho_\lambda + \tau_\lambda + \alpha_\lambda = 1$$

Particular cases:

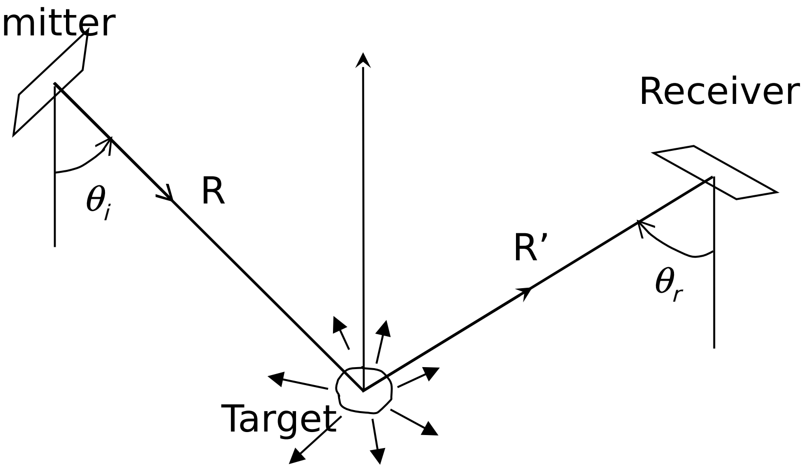
Black body: $\rho = \tau = 0 \qquad \alpha = 1$

Opaque body: $\tau = 0 \qquad \alpha + \rho = 1$

Kirchoff law: $\alpha = \varepsilon$ $\Rightarrow$ Black body: $\varepsilon = \alpha = 1$

(thermodynamical equilibrium) Opque body: $\varepsilon + \rho = 1$

The RADAR equation



ransmitted power by the radar:

$$P_i = \frac{P_e G_e}{4\pi} d\Omega$$

eceived irradiance at distance R:

$$E_i = \frac{P_e G_e}{4\pi R^2}$$

tercepted power by the target: $P_s = \frac{P_e G_e}{4\pi R^2} RCS$

Radar Cross Section (m²)

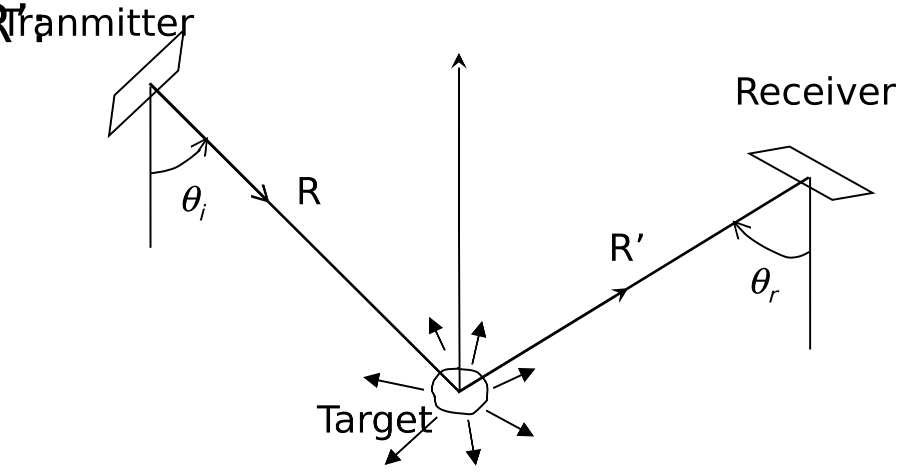
lected intensity by the target (cons. isotrope): $I = \frac{P_S}{4\pi} = \frac{P_e G_e}{4\pi R^2} \frac{RCS}{4\pi}$

ceived power by the surface dS at distance R’: $P_r = I d\Omega = I \frac{dS}{R'^2} = \frac{P_e G_e}{4\pi R^2} \frac{RCS}{4\pi R'^2} dS$

The RADAR equation (2)

Received power by dS at distance R':

$$P_r = \frac{P_e G_e}{4\pi R^2} \frac{RCS}{4\pi R'^2} dS$$



Received irradiance at distance R':

$$E_r = \frac{P_e G_e}{4\pi R^2} \frac{RCS}{4\pi R'^2}$$

Received power by the antenna:

$$P_r = E_r \, dA = E_r \frac{G_r \lambda^2}{4\pi} = \frac{P_e G_e}{4\pi R^2} \frac{RCS}{4\pi R'^2} \frac{G_r \lambda^2}{4\pi}$$

The RADAR equation (3)

eceived power by the antenna: $dP_r = \frac{P_e G_e}{4\pi R^2} \frac{RCS}{4\pi} \frac{G_r \lambda^2}{4\pi R^2}$

Case of surfaces:

Backscattering Radar Coefficient: $\sigma^0 = \frac{SER}{d\Sigma}$ (m²/m²)

$$dP_r = \frac{P_e G_e}{4\pi R^2} \frac{\sigma^0 d\Sigma}{4\pi} \frac{G_r \lambda^2}{4\pi R^2}$$

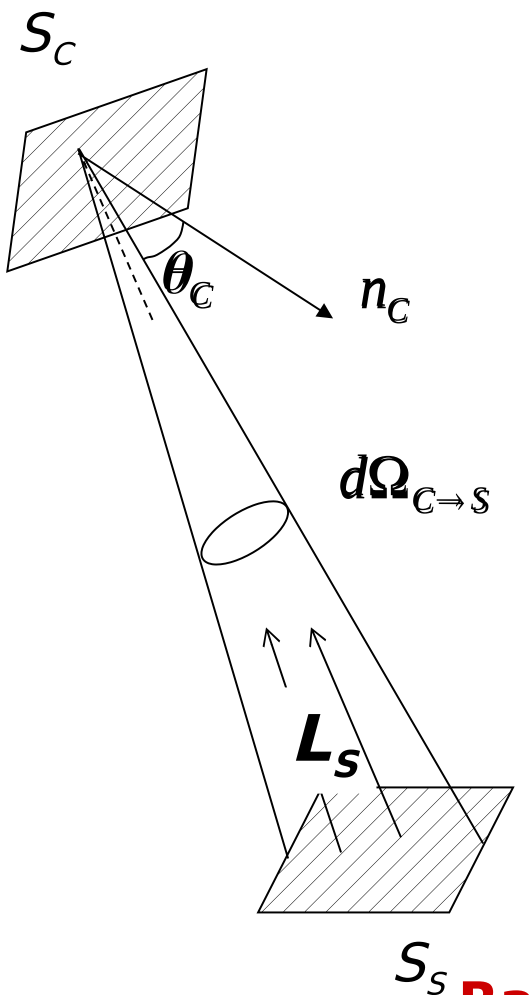
$$\langle P_r \rangle = \frac{\lambda^2}{(4\pi)^3} \frac{P_e \sigma^0}{R^4} \iint_{Surf.obs.} G_e G_r \, d\Sigma$$

ce characteristic measured by a sensor

System parameters

Measured power:

$$\Phi = L_S \cos\theta_C \, S_C \, \Omega_{C \rightarrow S}$$



==> *estimation de* L_s

Optics:

reflectance $\rho_b = \dfrac{\pi L_r}{E_i}$

IR Therm. & passive µwaves :

Brigthness Temperature: $T_b = \dfrac{2ckL_\lambda}{\lambda^4} = \varepsilon_\lambda T$

Radar:

Radar Backscattering Coefficient: $\sigma^0 \propto \rho_b = \dfrac{\pi L_r}{E_i}$